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13. ABSTRACT (Maximum 200 words)

In stochastic control, a major focus of this research was numerical methods for finding approximately optimal control laws. Dynamic programming and Monte Carlo optimization algorithms were followed. Both probabilistic methods, based on weak convergence ideas, and analytical methods were used to prove convergence of algorithms. The latter were based on viscosity solution methods for nonlinear partial differential equations. In nonlinear estimation, low dimensional approximate nonlinear filters were found for cases when a piecewise one-to-one function of a system state plus low intensity observation noise was observed.

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Final Technical Report

Grant No. AFOSR - 89-0015

W. H. Fleming and H. J. Kushner, Principal Investigators

During the period of this grant, October 1, 1988 – March 31, 1992

The Principal Investigators worked on several topics in stochastic control, nonlinear estimation and related areas.

W. H. Fleming

Progress was made by Fleming and associates on topics in stochastic control, theory of viscosity solutions of nonlinear partial differential equations, asymptotic problems for Markov diffusion processes and nonlinear filtering.

1. *Stochastic control.* Fleming and Fitzpatrick studied finite difference approximations to the dynamic programming equation for some particular classes of optimal stochastic control problems. These arise in models with two types of control actions. One represents a consumption rate, the other a rate of investment or exploration for an undiscovered natural resource (e.g. oil). These models have special linear-concave properties, which were used to prove convergence of optimal feedback control policies as the mesh size tends to zero. In a different direction is the Ph.D. research of H. Zhu on singular stochastic control problems. One part of Zhu's thesis extends earlier work of Fleming and Vermes; using convex duality techniques. Other parts of the thesis exploit viscosity solution methods to study a broad class of singular stochastic control problems.

2. *Viscosity solutions.* In earlier work, Fleming and Souganidis used viscosity solution methods to study asymptotic problems (such as the exit problem) for nearly deterministic controlled Markov diffusions. In a 1989 paper, they considered two-controller, zero-sum stochastic differential games,

and characterized the value function in terms of viscosity solutions. Recently, Fleming and McEneaney used viscosity solution methods and differential game ideas to study risk sensitive control of nonlinear systems.

3. *Asymptotic problems for Markov processes.* Fleming and James obtained much sharper versions of the Freidlin–Wentzell large deviations formulas for exit probabilities for nearly deterministic Markov diffusion processes. These results were obtained by a mixture of ideas from probability, partial differential equations and calculus of variations.

4. *Nonlinear filtering with small observation noise.* Fleming and Pardoux developed a technique which provides low-dimensional approximations to optimal filters (such as extended Kalman filters) in cases where a piecewise one-to-one function of a system state plus low intensity white noise is observed. This direction of work was continued by Fleming, Zhang and others. They dealt with sequential hypothesis tests, which are superior to the fixed time statistical tests proposed by Fleming and Pardoux, as well as numerical aspects of the problem.

Publications

1. *Convex duality approach to the optimal control of diffusions* (with D. Vermes), SIAM J. on Control and Optimization 27 (1989) 876–907.

2. *Value functions for two-player, zero-sum stochastic differential games* (with P. E. Souganidis), Indiana University Math J. 38 (1989) 293–312.

3. *Asymptotic expansions for Markov processes with levy generators* (with H. M. Soner), Applied Math. and Optimiz. 19 (1989) 203–223.

4. *Generalized solutions and convex duality in optimal control in partial differential equations and the calculus of variations* (ed F. Colombini et al), Birkhauser, 1989, 461–472.

5. *Piecewise monotone filtering with small observation noise*, (with E. Pardoux) SIAM J. on Control and Optimiz. 27 (1989) 1156–1181.

6. *Piecewise monotone filtering in discrete time with small observation noise*, (with D. Ji, P. Salame and Q. Zhang) IEEE Trans. Auto. Control 36 (1991) 1181–1185.

7. *Numerical method for an optimal investment/consumption problem*, (with B. Fitzpatrick) Math. Oper. Res. 16 (1991) 823–841.

8. *Asymptotic series and exit time probabilities*, (with M. James) to appear in Annals of Probability.

9. *Nonlinear filtering with small observation noise: piecewise monotone observations*, (with Q. Zhang) in Stochastic Analysis, eds. E. Merzbach, A. Schwartz and E. Mayer-Wolf. Academic Press (1991) 153–168.

10. *Piecewise monotone filtering with small observation noise* (with Q. Zhang) Proc. Joint US - France Workshop on Stochastic Analysis (eds. I. Karatzas and D. Ocone) April 1991.

11. *Risk sensitive optimal control and differential games* (with w. McEneaney), LCDS Report 92-1.

12. *Controlled Markov Processes and Viscosity Solutions* (with H. M. Soner) Springer - Verlag, 1992.

H. J. Kushner

A great deal of work was done in the area of numerical methods for stochastic control problems. Among the most challenging problems in the stochastic control area are the so-called singular control problems: They arise in numerous areas; for example, routing questions in computer and communications systems. We developed a powerful class of numerical methods for such problems. The basic models used are diffusions or reflected diffusions, but the method is a general applicability. The central idea is that of the Markov chain approximation method, where an approximation to the control problem is found for which an optimal solution is computable, and which is an arbitrarily good approximation to the original problem and its optimal value function. The methods are convenient to program and use (and they have been used with success), and they cover a wide variety of problems. In fact, for the singular problem, they seem to be the only ones currently available. Owing to problems in proving tightness of certain processes which occur in the convergence proofs, the methods of proof used for the non-singular problems need modifications. Examples of useful approximations, the algorithms and the convergence proofs are given. In order to illustrate the power of the methods, two classes of problems are dealt with; the first is a class of discounted problems, and the second is an average cost per unit time problem subject to some constraints, and which arises in the study of multi-customer class queueing networks under conditions of heavy traffic. The method is applicable to the more standard singular control and ergodic problems and with greater ease. The algorithms have been coded and tested on problems of practical interest. They work well.

Other work included advances in the use of Monte Carlo type methods for nonlinear systems modelled by diffusion type processes and where the dimension might be high. Here one parmetrizes the control a priori, and seeks to optimize the parameters, for the appropriate given cost of performance function. Often the performance function is not "nice;" for example, it might be the probability that an object leaves a region in a given time interval. This function is not nice since it is the expectation of an indicator function, which cannot be differentiated. A major problem in Monte-Carlo work is the estimation of the derivatives (with respect to the parameters) of the cost function from simulated samples. Methods for doing this were developed, based on using sensitivity functions (measure transformations). The method is asymptotically consistent, and yields good numerical results. The ergodic cost problem was also dealt with, a particularly hard problem in Monte-Carlo.

Publications

1. *Diffusion approximations and nearly optimal maintenance policies for system breakdown and repair policies* App. Math. and Optim. 20 (1989) 30–53.

2. *Stochastic approximation and large deviations: General results for w.p. 1 convergence*, sub. to SIAM J on Control and Optimization, LCDS Dept 87-21, 1987, SIAM J. on Control and Optimization, 27, 1989, 1108–1135.

3. *Numerical methods for stochastic control problems in continuous time*, SIAM J. on Control and Optimization, 28 (1990) 999-1048.

4. *Approximations and optimal control for the pathwise average cost per unit time and discounted problems for wideband noise driven systems*, SIAM J. on Control and Optimization 27 (1989) 546–562.

5. *Weak Convergence Methods and Singularly Perturbed Stochastic Control and Filtering Problems*, 1990, Birkhauser.

6. *A Monte-Carlo method for sensitivity analysis and parametric optimization of nonlinear stochastic systems; the ergodic case*, SIAM J. on Control and Optimization 30 (1992) 440–646.

7. *Numerical methods for stochastic singular control problems*, SIAM J. on Control and Optimization 29 (1991) 1443–1475.

8. *Limit theorems for pathwise average cost per unit time problems for controlled queues in heavy traffic*, to appear, Stochastics.

9. *A surrogate estimation approach for adaptive routing in communication networks*, sub. to IEEE Trans. on Automatic Control.

10. *Estimation of the derivative of an invariant measure with respect to a control parameter; applications to Monte-Carlo optimization*; to appear J. Applied Probability, 1992.